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SELFDECOMPOSABILITY AND S-SELFDECOMPOSABLITY: a view towards simulations?

ABSTRACT. We will present two classes of limiting distributions obtained from *sequences* (X_n) *of stochastically independent variables.*

The first one, is **the Lévy class L** of *selfdecomposable random variables* Z. It is defined as the class of all possible weak limits in the following scheme:

$$a_n(X_1 + X_2 + ... + X_n) + b_n \Rightarrow Z, \text{ as } n \to \infty, \tag{1}$$

where $a_n > 0$, and the triangular array $(a_n X_k, 1 \leq k \leq n; n \geq 1)$, is infinitesimal. [If (X_n) are vectors, constants a_n are replaced by matrices A_n.]

Class L is quite large and includes all stable variables, gamma variables, Student t-distribution, log-gamma, F distribution, Bessel-K and some related to Ising models. Each selfdecomposable Z admits the random integral representation:

$$Z = \int_0^\infty e^{-t} dY(t), \text{ for unique Background Driving Lévy Process Y (BDLP)}$$

That representation maybe used for simulation Z via BDLP.

The second class, is **the class** $\mathcal{U}$ of *s-selfdecomposable random variables.* These are the weak limits V in the following scheme: for stochastically independent variables (X_n) and constants $r_n > 0$, $b_n \in \mathbb{R}$,

$$U_{r_n}(X_1) + U_{r_n}(X_2) + ... + U_{r_n}(X_n) + b_n \Rightarrow V, \quad \text{as } n \to \infty, \tag{2}$$

where $U_r(x) := \max(|x| - r, 0) \frac{x}{|x|}, x \in \mathbb{R}$ are called *shrinking operations* (in short: s-operations) and array $U_{r_n}(X_k), k = 1, 2..., n$ is infinitesimal.

The s-operations are non-linear and form a semigroup $U_r(U_s(x)) = U_{r+s}(x)$. They were proposed by Kazimierz Urbanik around 1970. [Nowadays, they maybe interpreted as *European call option* in mathematical finance.]

Class $\mathcal{U}$ variables V admit the following random integral representation:

$$V = \int_0^1 t\, dY(t), \quad \text{for a unique Lévy process } Y.$$

The following inclusions hold:

(Gaussian) $\subset$ **(stable)** $\subset L \subset \mathcal{U} \subset ID$ **(infinitely divisible)**