A PDE APPROACH TO REGULARIZATION IN DEEP LEARNING

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The fundamental tool for training deep neural networks is Stochastic Gradient Descent applied to the loss function, \( f(x) \), which is high dimensional and nonconvex.

\[
\begin{align*}
\frac{dx_t}{dt} &= -\nabla f(x_t) dt + \sqrt{\beta^{-1}} dW_t \\
\end{align*}
\]

In this talk we discuss a modification of (SGD) which significantly improves the training time as well as the generalization error [COO+17]. We also discuss a related algorithm also allows for effective training of DNNs in parallel [CBZ+17].

The algorithm is based on [CCS+16], which replaced \( f \) in (SGD) with \( f_\gamma(x) \), the local entropy of \( f \), which is defined using notions from statistical physics [BBC+16].

We show that the local entropy is the solution of a Hamilton-Jacobi equation.

\[
\begin{align*}
\frac{dx_t}{dt} &= -\nabla u(x,T-t) + \sqrt{\beta^{-1}} dW_t, \quad 0 \leq t \leq T \\
\end{align*}
\]

where \( T \) is a fixed time horizon, and \( u(x,t) \) is the solution of initial value problem for the viscous Hamilton-Jacobi PDE

\[
\begin{align*}
\frac{\partial u(x,t)}{\partial t} + \frac{1}{2} |\nabla u(x,t)|^2 &= \frac{\beta^{-1}}{2} \Delta u(x,t), \quad 0 \leq t \leq T \\
\end{align*}
\]

with initial data \( u(x,0) = f(x) \).

The gradient \( \nabla u(x,t) \) can be computed using Langevin MCMC, by solving an auxiliary SGD equation.

Using the stochastic control interpretation of a slightly modified evolution, we prove that the expected value of the loss function is lower compared to (SGD).

REFERENCES


